Vulnerability Level 2 Criterion for Parametric Roll

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The procedure described for vulnerability level II criterion for parametric roll addresses some of the considerations identified in Bassler, et al. (2009) and uses the framework outlined in Belenky, et al. (2009).

Observation of the largest absolute value of the roll angle observed during the wave group pass is the objective and can be used as a criterion. The initial conditions for the numerical solution of (12) can be chosen as 5-10 degrees of initial roll angle and zero roll velocity to assure if parametric resonance is possible while the wave group passes the ship.

Description of the Method

Vulnerability to parametric roll is determined by the maximum angle of roll response on a "typical" wave group related to a given sea state.

Formulation for a Wave Group

The "typical" wave group, shown in Figure 1, is assumed to consist from a number of waves of the same length and the period corresponding to the spectral mean period. Justification of this assumption is considered in a following section:

$$\zeta(t) = A(t)\cos(\omega_1 t) \qquad (1)$$

where ω_1 is the mean frequency, A(t) is an amplitude of the group; it is defined with an sine function envelope:

$$A(t) = A_{\min} + 0.5(A_{\max} - A_{\min})\sin(\omega_A t - 0.5\pi) \qquad (2)$$

$A_{\min}$ and $A_{\max}$ are the minimum and maximum amplitude of the group, respectively. ω_A is an envelope frequency defined as:

$$\omega_A = \frac{2\pi}{T_G} \qquad (3)$$

T_G is a time interval for a group to pass a fixed point, and depends on number of waves in a group and the mean period:

$$T_G = N_G T_1 \qquad (4)$$

where N_G is assumed number of waves in group. The amplitude of the group is considered as a function of time only; its spatial change is not modeled.

For simplicity, consideration of the wave direction is limited to only head or following seas. Encounter frequency is expressed as:

$$\omega_e = \omega_1 + k_{dir} k_1 V_S \qquad (5)$$

where V_S is forward speed, k_1 is the wave number corresponding to the mean period, T_1,and k_{dir} is a wave direction coefficient; it equals 1 for head seas and -1 for following seas.

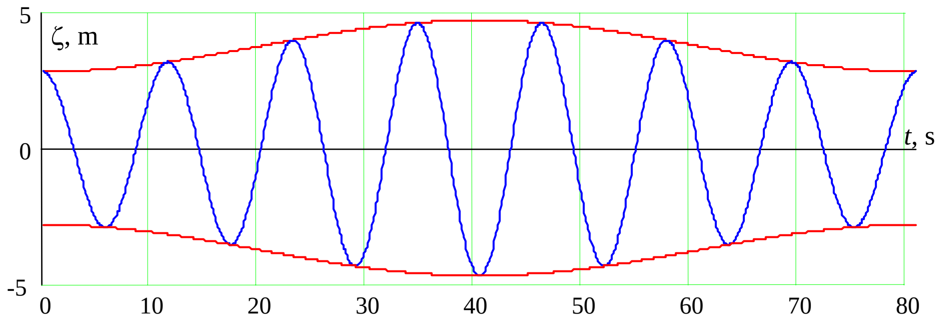

Figure 1. Time history of a wave group passing fixed point in space, sea state 7

The time while the group passes a point at midship section is expressed as:

$$T_{Ge} = \frac{N_G \lambda_1}{c + k_{dir} Vs} \qquad (6)$$

where λ_1 is the wave length corresponding to the mean period, and c is the wave celerity. The relation between period, length, and wave celerity is:

$$T_1 = \frac{\lambda_1}{c} \qquad (7)$$

The dispersion relation is used to connect the mean frequency to the wave length (through the wave number):

$$\lambda_1 = \frac{2\pi}{k_1}; \qquad k_1 = \frac{\omega_1^2}{g} \qquad (9)$$

The change in the time while the wave group passes the midship section affects the formula for the amplitude envelope:

$$A_e(t) = A_{\min} + 0.5(A_{\max} - A_{\min})\sin(\omega_{Ae} t - 0.5\pi); \quad \omega_{Ae} = \frac{2\pi}{T_{Ge}} \qquad (10)$$

This leads to a re-formulation of the group description in terms of the frequency of encounter:

$$\zeta_e(t) = A_e(t)\cos(\omega_e t) \qquad (11)$$

Formulation for the Roll Response

The roll response is described using a dynamical system with one degree of freedom, linear damping, and time-dependent stiffness:

$$\ddot{\phi} + 2\delta_\phi \dot{\phi} + \omega_{\phi 0}^2 f_\phi(\phi, t) = 0 \qquad (12)$$

where δ_ϕ is a linear coefficient of roll damping, $\omega_{\phi 0}$ is a natural roll frequency, and $f_\phi(\phi,t)$ is the time-dependent stiffness. The roll response is modeled using

$$f_\phi(\phi,t) = \frac{GZ(\phi,t)}{GM_0}; \quad GZ(\phi,t) = GZ_0(\phi) - \left(GM_0 - GM(t)\right)\sin(\phi) \tag{13}$$

where $GZ_0(\phi)$ is the stability curve for calm water, and GM_0 refers to calm water GZ curve, and $GM(t)$ is the time varying GM due to passage of the wave group.

The dynamical system (12) is described by a homogeneous nonlinear ordinary differential equation with parametric excitation. It is solved numerically using a conventional Runge-Kutta method. In order to observe parametric resonance while the wave group passes, there should be non-zero initial conditions.

Evaluation of Stability Changes due to Passage of the Wave Group

Because the criterion is intended to distinguish an unconventional ship, the waterline geometry may change dramatically as a wave groups passes the vessel. In order to produce representative stability changes, the ship waterline has to be presented with maximum accuracy limited by the reasonable complexity of the calculation method.

This proposal evaluates the waterline and accounts for pitch and heave motions that may be induced by the passing wave group. The following equations are used to describe these motions:

$$\begin{cases} (M + A_{33})\ddot{\zeta}_G + B_{33}\dot{\zeta}_G + F_\zeta(\zeta_G,\theta,t) = 0 \\ (I_Y + A_{55})\ddot{\theta} + B_{55}\dot{\theta} + M_\theta(\zeta_G,\theta,t) = 0 \end{cases} \tag{14}$$

where M is mass of the ship, I_Y is mass moment of inertia relative transversal axes, A_{33} and A_{55} are heave added mass and pitch moment of inertia, respectively; and B_{33} and B_{55} are damping coefficients for heave and pitch. Functions F_ζ and M_θ are the difference between Froude-Krylov and hydrostatic forces and moments at the instant of time, t. These values are expressed as follows:

$$F_\zeta(\zeta_G,\theta,t) = \rho g\left(V_0 - \int_{-0.5L}^{0.5L}\Omega(x, z(\zeta_G,\theta,t))dx\right) \tag{15}$$

$$M_\theta(\zeta_G,\theta,t) = \rho g\left(V_0 \cdot LCB_0 - \int_{-0.5L}^{0.5L}M_\Omega(x, z(\zeta_G,\theta,t))dx\right) \tag{16}$$

where ρ is mass density of water, V_0 volumetric displacement in calm water, LCB_0 is the longitudinal position of center of buoyancy in calm water. Functions Ω and M_Ω calculate an area and a static moment relative to y-axes of a station located at abscissa x. The second argument of this function shows submergence of this station is expressed by the function of instantaneous waterline $z(\zeta_G,\theta,t)$. The instantaneous waterline function depends on heave displacement, pitch angle, and current time.

To simplify the calculation, the following assumptions were made for added mass in heave and pitch:

$$A_{33} = M; \quad A_{55} = I_Y \tag{17}$$

Damping for heave and pitch is expressed as a fraction of critical damping:

$$B_{33} = \mu_h \omega_{\zeta 0}(M + A_{33}); \quad B_{55} = \mu_p \omega_{\theta 0}(I_Y + A_{55}) \tag{18}$$

where $\omega_{\zeta 0}$ and $\omega_{\theta 0}$ are the natural frequency of heave and pitch, respectively, and can be expressed as follows:

$$\omega_{\zeta 0} = \sqrt{\frac{\rho g S_{WL0}}{M + A_{33}}}\,; \quad \omega_{\theta 0} = \sqrt{\frac{\rho g M \cdot GM_{L0}}{M + A_{33}}} \tag{19}$$

where S_{WL0} is the area of the waterplane and GM_{L0} is the longitudinal value of *GM* in calm water.

The system of equations (14) is solved numerically; and yields heave and pitch responses from the passing wave group and allows evaluation of the time-history of the instantaneous ship waterline. Once the instantaneous waterlines are available, instantaneous $GM(t)$ can be calculated and further used in formula (13).

The numerical solution of the system of equations (14) requires initial conditions. The initial conditions were taken from the steady-state solution of (14), under the action of a regular wave corresponding to the beginning of the wave group. This approach was used in order to avoid excessive motions due to initial transition. Then, the only transient behavior observed in (14) is related to the change in excitation amplitude.

Choice of the Characteristics of the Wave Group

It is assumed that all the waves in the group have the same frequency, equal to the mean frequency of the spectrum. This assumption can be justified by the form of the joint distribution of amplitudes and derivatives of phases. The derivative of the phases plays the role of random frequency in envelope presentation. The joint distribution is shown in Figure 2 and equation (20).

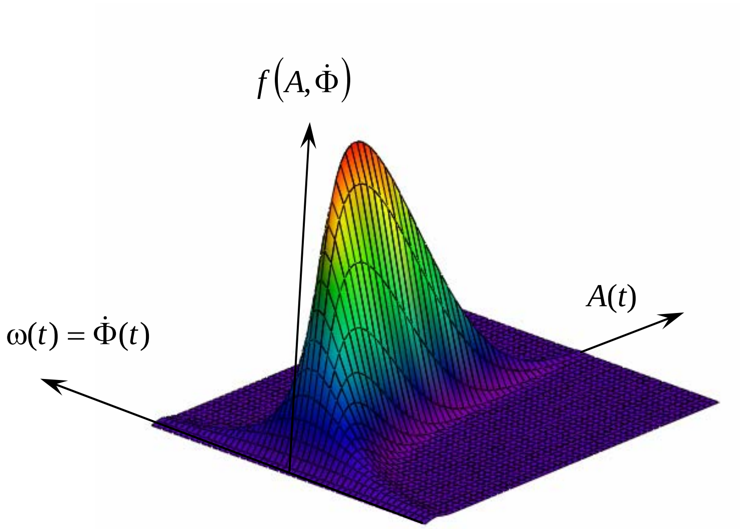

Figure 2. Joint distribution of amplitudes and derivatives of phases for the envelope presentation

$$f(A,\dot{\Phi}) = \frac{A^2}{\sigma_\zeta^3\sqrt{2\pi}\sqrt{\omega_2^2-\omega_1^2}}\exp\left(-A^2\cdot\frac{\omega_2^2-2\omega_1\dot{\Phi}+\dot{\Phi}^2}{2\sigma_\zeta^2(\omega_2^2-\omega_1^2)}\right) \qquad (20)$$

where σ_ζ is the standard deviation of wave elevations, $\dot{\Phi}$ is the derivation of the phase of the envelope that is usually considered as a random frequency, and ω_2 is the average width of a spectrum:

$$\omega_2 = \frac{1}{\sigma_\zeta}\sqrt{\int_0^\infty s(\omega)\omega^2 d\omega} \qquad (21)$$

For the envelope presentation, the amplitude follows Rayleigh distribution, and the conditional PDF, its mean value and variance for given amplitude value, can be expressed as:

$$f(\dot{\Phi}\,|\,A) = \frac{f(A,\dot{\Phi})}{f(A)};\quad m(\dot{\Phi}\,|\,A) = \int_{-\infty}^{\infty} f(\dot{\Phi}\,|\,A)\dot{\Phi}d\dot{\Phi} = \omega_1$$
$$V(\dot{\Phi}\,|\,A) = \int_{-\infty}^{\infty} f(\dot{\Phi}\,|\,A)(\dot{\Phi}-m(\dot{\Phi}\,|\,A))^2 d\dot{\Phi} \qquad (22)$$

The dependence of conditional variance is shown in Figure 3. It is clear that increasing amplitude results in decreased variance. Because the conditional mean value is equal to the mean frequency of the spectrum, and the variance for the frequency of large waves becomes small, then the larger waves are very likely to have a mean frequency. The wave group is meant to consist of large waves, and this justifies the choice of the frequency of the wave group.

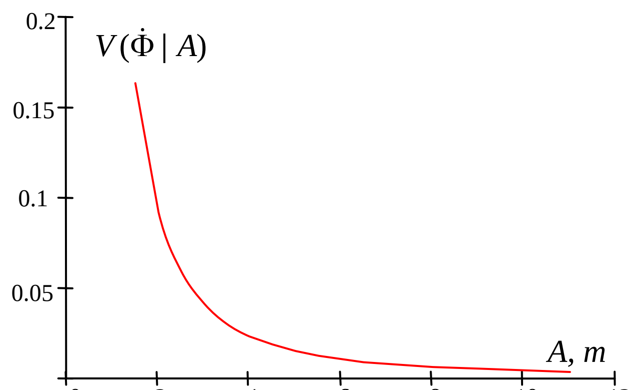

Figure 3. Conditional variance of derivatives of phases as a function of amplitudes for Bretschneider spectrum at sea state 8

Three more parameters are left to define: the number of waves in a group, and the initial and maximum amplitude. A robust choice of these parameters can be made based on wave statistics, either measured or simulated. For the testing purposes, the following values of these parameters were chosen:

$$N_G = 7;\quad A_{\min} = 0.5H_S;\quad A_{\max} = 1.5A_{\min} \qquad (23)$$

where H_S is significant wave height.

Sample Calculations

Example of Calculation for C11-class Containership

The lines of a sample containership are shown in Figure 4 and the principal particulars can be found in Table 1. The calculations were done for sea state 8 (Bretscheider spectrum, significant wave height 11.5 m and modal period 16.4 s). The heave and pitch responses during the passage of the wave group are presented in Figure 5 and were calculated for assumed heave and pitch damping of 4% of critical. Speed was chosen to correspond as closely as possible to the principle parametric resonance condition:

$$Vs = \begin{cases} V_d = |2\omega_{\phi 0} - \omega_1| \, k_1^{-1} & if \quad V_d < V_{\max} \\ V_{\max} & \end{cases} \tag{24}$$

where V_{max} is maximal available speed for the given sea state.

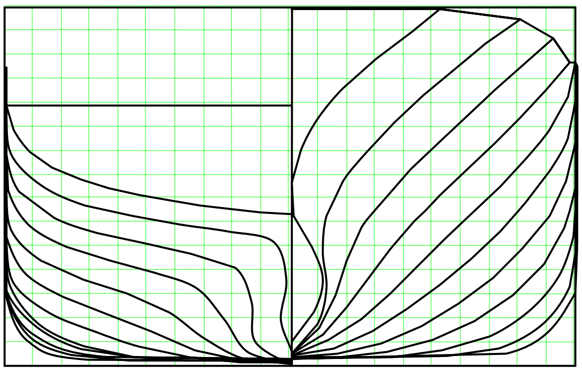

Figure 4. Geometry of the C11-class containership

Table 1. Principal Particulars – C11

Length, BP, m	262
Breadth, B, m	49
Depth, D, m	24.7
Draft, d, m	12.83
GM value, m	1.91

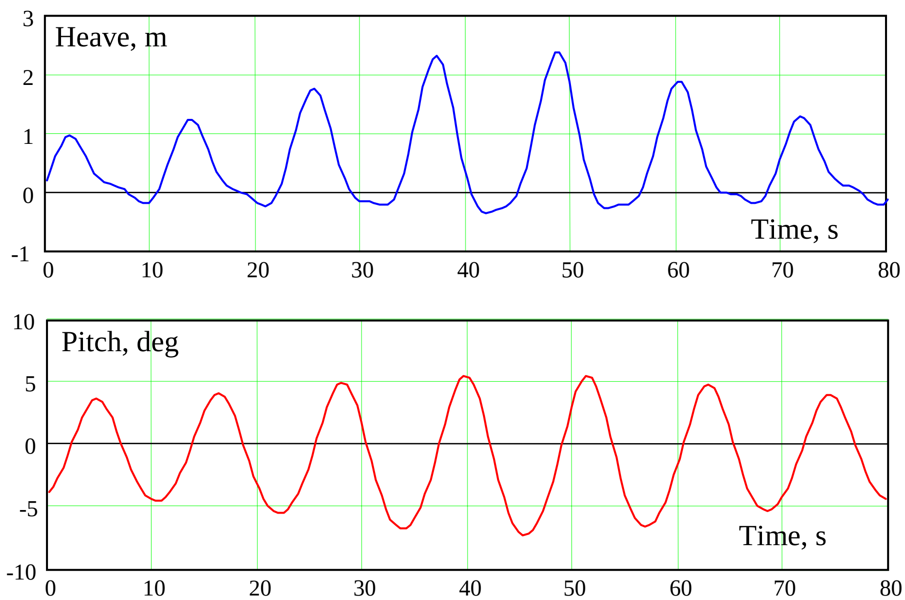

Figure 5. Heave and pitch response of C11-class containership to passing wave group

Once heave and pitch have been calculated, the instantaneous waterlines corresponding to the instantaneous attitude of the ship at each time step can be determined. Once the instantaneous waterlines are determined, the GM-value in waves can be evaluated (see Figure 6). Then, the stability in waves was modeled with formula (13) and the roll response was numerically evaluated (see Figure 7).

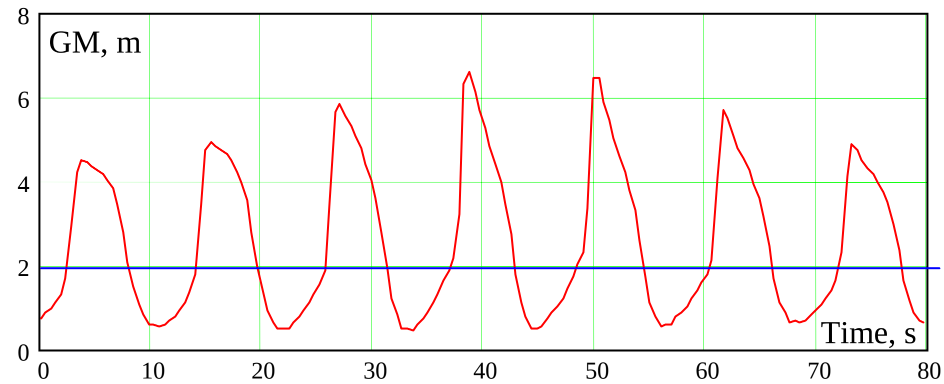

Figure 6. GM response of C11-class containership to a passing wave group

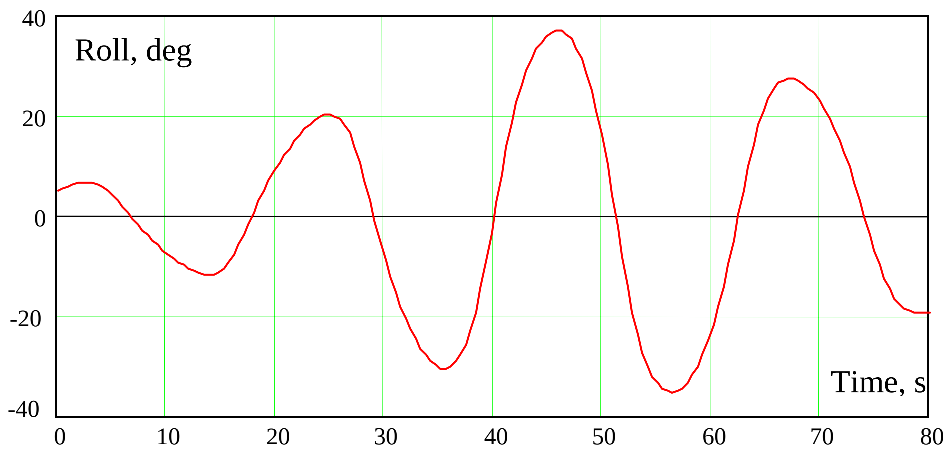

Figure 7. Roll response of C11-class containership to a passing wave group, μ_ϕ=0.04

Sample Application of Criterion

In addition to the C11-class containership, three other ships were used for sample application. The calculations were performed for sea states 5 through 8. The lines of these ships are shown in Figures 7 through 9 and the particulars are given in Tables 2 through 4. The results of calculations are given in Table 5.

For a given sea state, the initial conditions and maximum speed were varied depending on the type of vessel to account for normal operational conditions. Loading conditions were also assumed, which corresponded to normal operational conditions. To fully develop a criterion, choice of this parameter should be formalized.

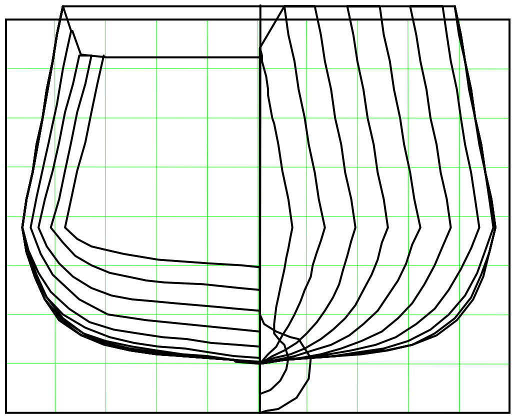

Figure 8. Geometry of the ONR tumblehome topside configuration

Table 2. Principal Particulars

Length, BP, m	154
Breadth, B, m	18.8
Depth, D, m	12.5
Draft, m	5.5
GM - value, m	2.42

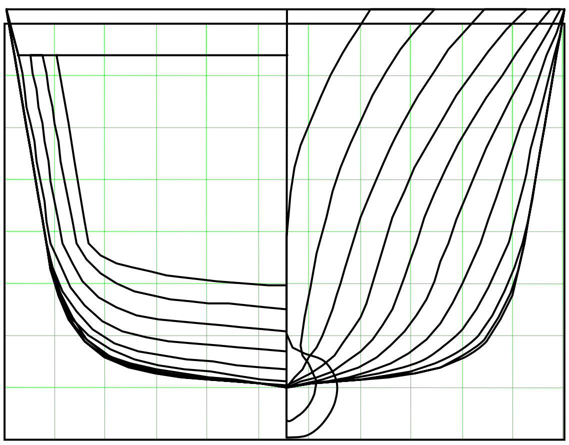

Figure 9. Geometry of the ONR flared topside configuration

Table 3. Principal Particulars ONR flared

Length, BP, m	154
Breadth, B, m	18.8
Depth, D, m	12.5
Draft, m	5.5
GM - value, m	0.9

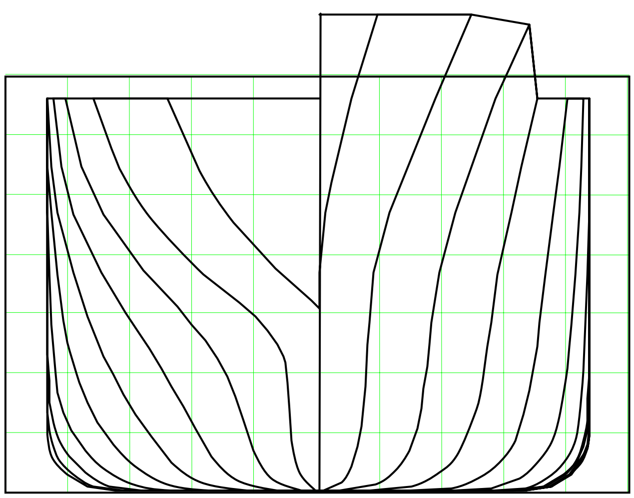

Figure 10. Geometry of the Series 60 C_B=0.7

Table 4. Principal Particulars – Series 60

Length, BP, m	121.92
Breadth, B, m	17.4
Depth, D, m	13.15
Draft, m	6.97
GM - value, m	1.0

Table 5. Results of sample calculations

***H_S*, m**	***T_m*, s**	**λ_1, m**	**Limiting Speed, kn**	**V_s, kn**	**Wave Heading**	**$\omega_{1e}/\omega_{\phi 0}$**	**Roll Damping**	**Max Roll, deg**
C11-class containership, *L*=262; $\phi_0 = 5$ deg; $\dot{\phi}_0 = 0$ deg/ *s*								
11.5	16.4	250	5	3.67	Head	2.0	0.03	39.5
							0.04	37.0
							0.05	34.8
							0.06	32.8
7.5	15	209.8	10	0.06	Head	2.0	0.03	187
							0.04	191
							0.05	187
							0.06	68.1
5	12.4	143.4	25	5	Following	2.0	0.03	5
							0.04	5
							0.05	5
							0.06	5
3.25	9.7	87.6	25	8	Following	2.0	0.03	7
							0.04	6
							0.05	5.4
							0.06	5
ONR Tumblehome Topside, *L*=152 m $\phi_0 = 10$ deg; $\dot{\phi}_0 = 0$ deg/ *s*								
11.5	16.4	250	10	10	Head	0.962	0.05	10
							0.10	10
							0.15	10
							0.20	10
7.5	15	209.8	15	15	Head	1.19	0.05	10
							0.1	10
							0.15	10
							0.2	10
5	12.4	143.4	30	28.5	Head	2.0	0.05	10.2
							0.10	10
							0.15	10
							0.20	10
3.25	9.7	87.6	30	12.5	Head	2.0	0.05	10
							0.1	10
							0.15	10
							0.2	10

Table 5 Results of sample calculations (cont.)

***H**$_S$*, m	***T**$_m$*, s	λ_1, m	**Limiting Speed, kn**	**V$_s$, kn**	**Wave Heading**	$\omega_{1e}/\omega_{\phi 0}$	**Roll Damping**	**Max Roll, deg**
ONR Flared Topside, L=152 m $\phi_0 = 10$ deg; $\dot{\phi}_0 = 0$ deg/ s								
11.5	16.4	250	10	10	Head	1.57	0.05	12.6
							0.1	11.0
							0.15	10
							0.20	10
7.5	15	209.8	15	15	Head	1.95	0.05	53.0
							0.1	25.5
							0.15	10.6
							0.2	10
5.0	12.4	143.4	25	6.1	Head	2.0	0.05	26.7
							0.10	11.0
							0.15	10
							0.2	10
3.25	9.7	87.6	25	1.2	Following	3.0	0.05	14.8
							0.1	10
							0.15	10
							0.2	10
Series 60 (C_B=0.7), L=121.92; $\phi_0 = 5$ deg; $\dot{\phi}_0 = 0$ deg/ s								
11.5	16.4	250	5	5	Head	1.24	0.03	5
							0.05	5
							0.08	5
							0.10	5
7.5	15	209.8	10	10	Head	1.54	0.03	5.8
							0.05	5
							0.08	5
							0.1	5
5	12.4	143.4	16	11	Head	2.0	0.03	5
							0.05	5
							0.08	5
							0.10	5
3.25	9.7	87.6	16	1.76	Head	2.0	0.03	5
							0.05	5
							0.08	5
							0.1	5

Summary and Concluding Comments

Vulnerability to parametric roll was determined using the maximum angle of ship roll response to a "typical" wave group, related to a given sea state.

The period of waves in a "typical" group can be assumed to be equal to the mean period. This can be justified using envelope theory, which shows that conditional variance of wave period dramatically decreases with growth of wave amplitude. Therefore, large waves are likely to have a period very close to the mean period.

Other characteristics of the "typical" group were chosen from practical considerations: the number of waves considered was equal to 7. It is known that development of parametric roll usually takes from 5 to 9 waves to develop. The height of the waves in the group changes within 50% of the significant wave height. The wave height for the initial wave in a group was considered equal to significant wave height. Values of these parameters may need further justification and refinement for increased robustness of application of the method.

The maximum roll angle was calculated using the numerical solution of single-degree-of-freedom roll equation with linear, or linearized, damping and nonlinear time-dependent stiffness. The latter is calculated by "modulation" of the calm water *GZ* curve with time-dependent *GM* value. The time-dependent *GM* value was calculated using the instantaneous waterplane.

The instantaneous waterplane was evaluated using the attitude of the ship in waves, accounting for heave and pitch response, for the ship encountering a "typical" wave group.

The example calculation included four ships. The calculations were performed for a series of roll damping coefficients typical for the respective ship type. Operational loading conditions were assumed. The speed was chosen to closely match the frequency of encounter for principle parametric resonance.

The C11-class containership is known for its vulnerability to parametric roll. The proposed criteria shows large roll angles for the C11-class encountering representative wave groups in sea state 7 and 8. As expected, Series 60, which is representative of a conventional ship type, did not show any vulnerability for the considered loading conditions.

Both ONR Topside configurations (flared and tumblehome) have relatively large bilge keels. The range of damping coefficients was meant to model both bare and fully appended hulls. While the ONR Tumblehome Topside did not show any parametric roll for the analyzed loading condition, parametric roll was observed for ONR Flared Topside, for small roll damping. Parametric roll was not observed for the flared configuration with roll damping coefficients corresponding to the fully appended hull. This is consistent with findings from earlier numerical and experimental investigations for these hull forms (Bassler, 2008; Olivieri, et al., 2008; Hashimoto and Matsuda, 2009).

The choice of the standard for this vulnerability criterion requires additional discussion. From preliminary observations, one can conclude that parametric roll can be seen somewhere between 20 to 30 degrees as maximum observed roll. The standard may be implemented with consideration for ship type, particularly related to machinery failure, cargo shift, or excessive risk of crew / passenger injury.

Formal procedures also need to be developed for setting roll damping and loading conditions. These procedures could also depend on ship type and /or ship size.

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